Chapter-3

Structures for FIR & IIR Systems.

Structures for realization of discrete time systems :-

We know the general linear constant-coeff diff eq. is

delay dependent fb coeff — delay dependent feed forward coeff &

$$y(n) = -\sum_{k=1}^{N} a_k y(n-k) + \sum_{k=0}^{M} b_k x(n-k) \quad \text{--- (1)}$$

& in terms of z-transform the LTI discrete time systems r char by rational system function

$$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 + \sum_{k=1}^{N} a_k z^{-k}} \quad \text{--- (2)}$$

Implementation of (1) & (2) have to be studied in h/w or in s/w on a programmable digital computer

—*—

→ Computational complexity

→ Memory requirements

→ Finite word length effects (finite precision effects) refers to quantization effects that r inherent in any digital implementation of the system. e.g. some values r truncated or rounded-off for computing an o/p to fit the limited precision constraint of the computer or the h/w used in implementation.

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Structure for FIR systems

∴ FIR system is described by diff eq:

$$y(n) = \sum_{k=0}^{M-1} b_k\, x(n-k)$$ (doesn't depend on past o/ps)

dependence: feed forward coeff ∴ $H(z) = \sum_{k=0}^{M-1} b_k z^{-k}$

& the unit sample response $(h(n))$ of FIR system is identical to the coeff $\{b_k\}$

i.e.

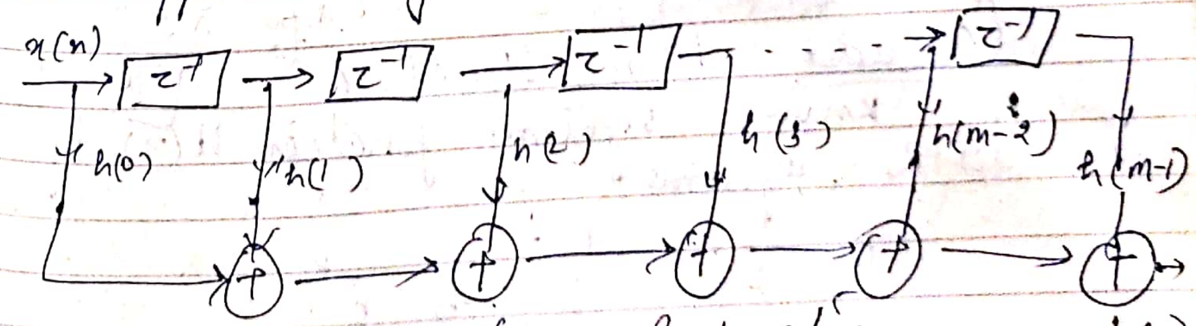
$$h(n) = \begin{cases} b_n, & 0 \le n \le M-1 \\ 0, & \text{otherwise} \end{cases}$$

(1) Direct-form structure:

For the non-recursive systems i.e. FIR. (i.e. present o/p is independent of past o/ps)

$$y(n) = \sum_{k=0}^{M-1} h(k)\, x(n-k)$$

which involve M-1 memory locations for storing M-1 previous i/ps & M multiplications & M-1 additions per o/p pt. ∴ o/p resembles a tapped delay line or a transversal system



Direct form Realization

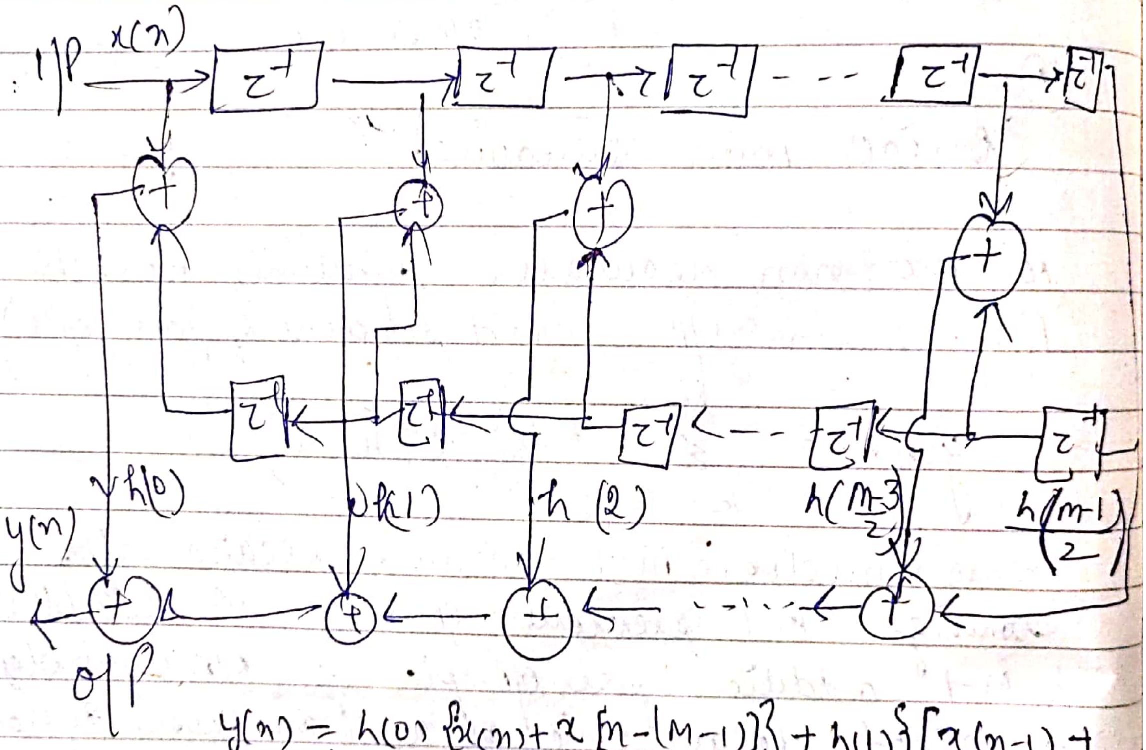
y(n)

for Linear-Phase FIR system, the unit sample response h(n) satisfies either symmetry or asymmetry condition i.e.

$$h(n) = \pm h(M-1-n)$$

here the no. of multiplications is reduced from M to M/2 for M even & to (M-1)/2 for M odd.

for M (odd)



$y(n) = h(0)\{x(n) + x[n-(M-1)]\} + h(1)\{[x(n-1) + x(n-(M-2))]\} + \dots h(\frac{M}{2}-1)\{x[n-(\frac{M}{2}-1)]$

Cascade - Form Structure

We know system function H(z) of FIR systems

$$H(z) = \sum_{k=0}^{M-1} b_k z^{-k}$$

factor $H(z)$ into 2nd order FIR system so that

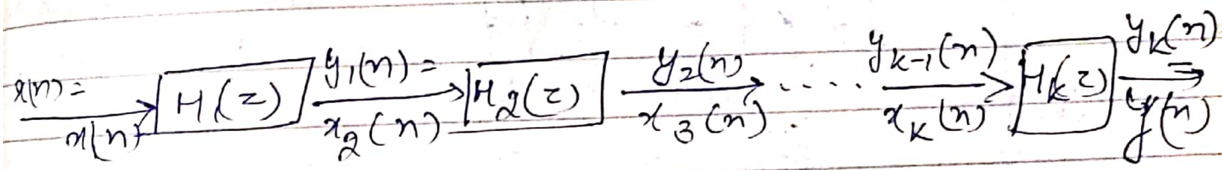
$$H(z) = \prod_{k=1}^{K} H_k(z)$$

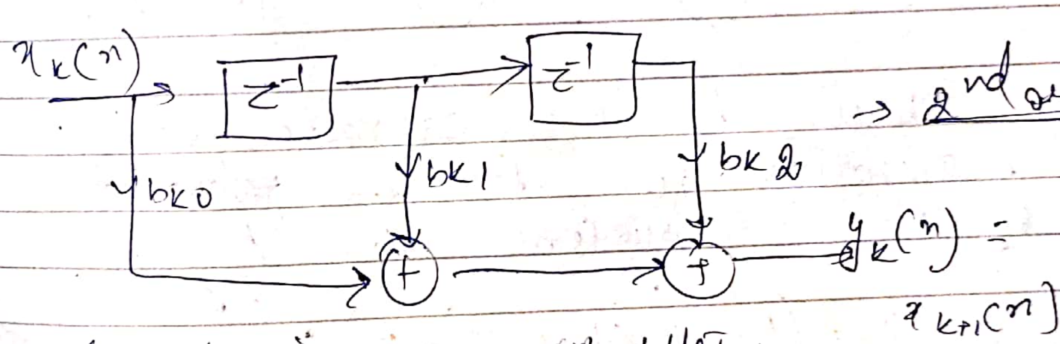
where, $H_k(z) = b_{k0} + b_{k1} z^{-1} + b_{k2} z^{-2}$

$k = 1, 2, \dots K.$

where K is the integer part of $\frac{M+1}{2}$.

The filter parameter b_0 may be equally distributed among K filter sections such that $b_0 = b_{10} b_{20} b_{30}, \dots b_{K0}$.





→ 2nd order

For linear phase FIR filter, the symmetry in $h(n)$ implies that the zeros of $H(z)$ also exhibits a form of symmetry. If z_k & z_k^* a pair of complex conjugate zeros then $\frac{1}{z_k}$ & $\frac{1}{z_k^*}$ also pair of complex conjugate zeros. Then 4th-order FIR system is

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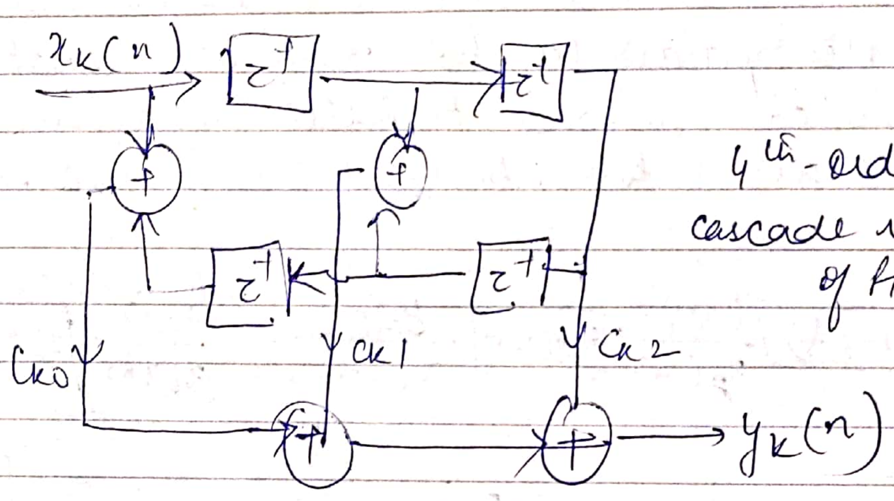
$$H_k(z) = c_{k0}(1 - z_k z^{-1})(1 - z_k^* z^{-1})\left(1 - z^{-1}\cdot\frac{1}{z_k}\right)\left(1 - \frac{z^{-1}}{z_k^*}\right)$$

$$= c_{k0} + c_{k1} z^{-1} + c_{k2} z^{-2} + c_{k1} z^{-3} + z^{-4}$$

where, $\{c_{k1}\}$ & $\{c_{k2}\}$ are functions of z_k.



4th-order cascade realization of FIR system

Q. Obtain direct & cascade form realization for transfer function of the FIR system given by

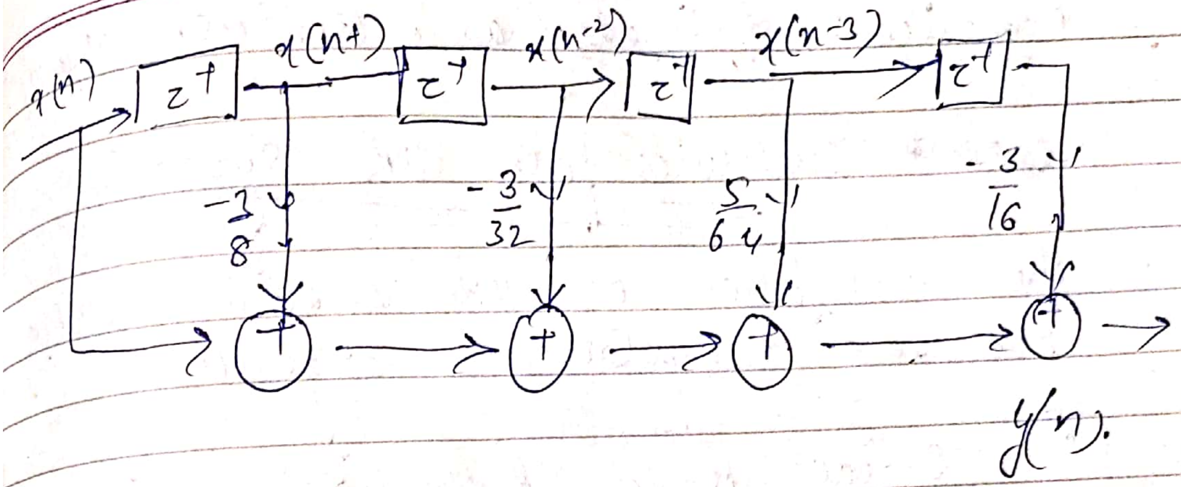
$$H(z) = \left(1 - \frac{1}{4}z^{-1} + \frac{3}{8}z^{-2}\right)\left(1 - \frac{1}{8}z^{-1} - \frac{1}{2}z^{-2}\right)$$

Solution

Direct form

$$H(z) = 1 - \frac{3}{8}z^{-1} - \frac{3}{32}z^{-2} + \frac{5}{64}z^{-3} - \frac{3}{16}z^{-4}$$

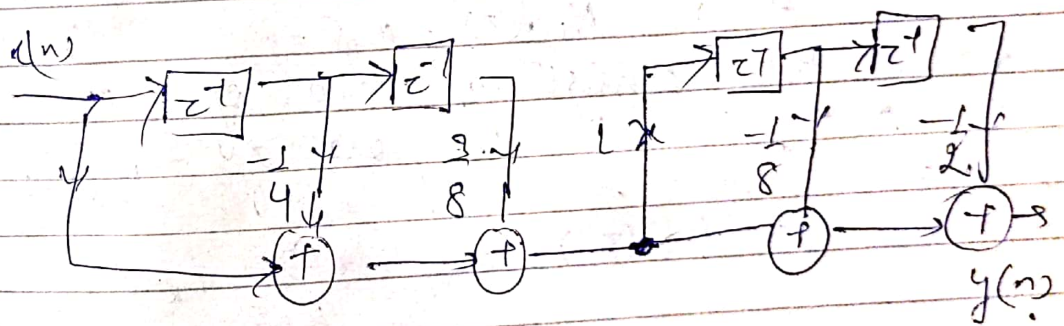
$$H(z) = \frac{Y(z)}{X(z)}$$



2nd – Cascade form.

$$H(z) = \left(1 - \frac{1}{4}z^{-1} + \frac{3}{8}z^{-2}\right)\left(1 - \frac{1}{8}z^{-1} - \frac{1}{2}z^{-2}\right)$$

$$= H_1(z) \cdot H_2(z)$$



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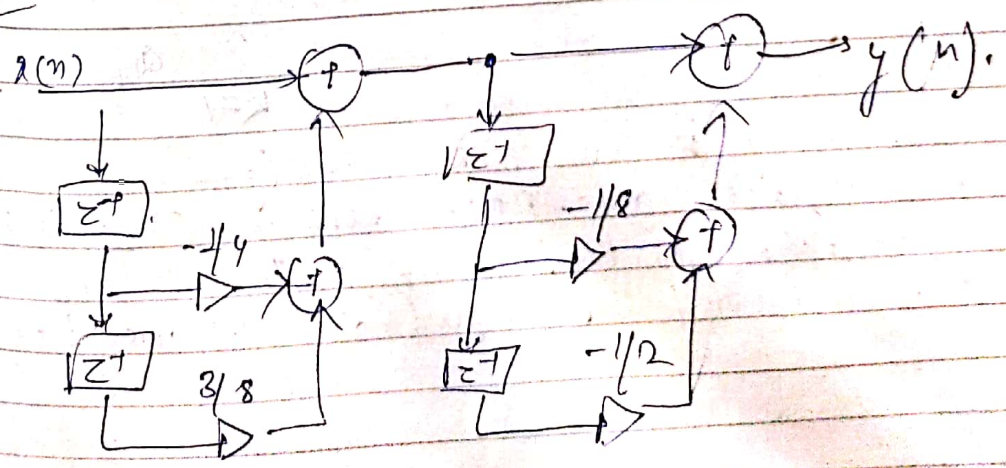

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